

Technology and the Yin&Yang of Teaching and Learning Mathematics

**The essence of using technology,
in particular computer algebra systems (CAS), in education**

Bernhard Kutzler
Linz, Austria

b@kutzer.com, <http://b.kutzer.com>

Abstract: We develop a model comprising six teaching and learning archetypes and use this model to look at the various roles that technology, in particular computer algebra systems (CAS), can play for each.

Table of Contents

Introduction	3
Mathematics/Pedagogy/Technology-Space	7
Represent	11
Document	19
Communicate	23
Compensate	26
Solve	33
Explore	39
More Thoughts about Teaching and Technology	47
Casanova or Don Juan?	49
References	51
Acknowledgement	52

Preface

This is a summary of my academic work of the past twenty years. I dedicate this text to two giants who let me stand on their shoulders: Bruno Buchberger and David Stoutemyer. Their influence on my work was enormous. Thank you!

Bernhard Kutzler, November 2008

Introduction

Humans are ruled by two forces: *Hold on* and *Let go*. These two forces correspond to *Yin* and *Yang*, the two elementary polar energies that are considered and studied in Eastern philosophies. In the context of teaching and learning mathematics and related subjects these two energies manifest as *Connect* and *Automate*. *Connecting* is an active, seeking form of *holding on*. *Automating* means to let a tool do what we used to do ourselves (such as performing arithmetic operations), i.e. we *let go* these tasks.

Hold on	-	Yin	-	Connect
Let go	-	Yang	-	Automate



A car is a tool for automating transportation. Instead of walking to the grocery shop, we can go there by car. This saves us from having to walk between our home and the shop and from having to carry the groceries. For some people, using a car for their shopping is a *convenience* that saves time and energy that they can then use for other activities – such as reading a book. For people who are physically challenged, using a car for their daily shopping may be a matter of *survival*.



This example shows two motivations for automation: *Amplification* and *compensation*. Here is another example: Optical instruments such as telescopes and microscopes *amplify* our natural eye sight so that we can see things that we cannot see otherwise. Optical instruments such as eye-glasses *compensate* poor eyesight so that people with poor eyesight can see things that people with normal eyesight can see without glasses.



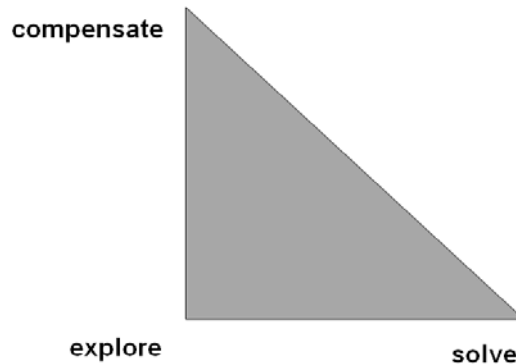
This can be further refined. *Amplification* in itself has two aspects based on the motivation to amplify. One can use a telescope to look at a distant object as may a private detective or a policeman do when observing a suspect – or as may an astronomer do when observing a moon eclipse. Alternatively one can use a telescope to scan the sky in the search for new stars. These two uses may be named *solving* and *exploring*. Likewise with the car example: Using a car for shopping or for visiting a friend who lives in another city *solves* a transportation problem.

Driving a car around California on a holiday trip is a nice way to *explore* the most populous U.S. state.

This gives three automation archetypes based on the motivation to automate:

$$\begin{aligned}\mathbf{Automate} &= \mathbf{Compensate} + \mathbf{Amplify} \\ &= \mathbf{Compensate} + \mathbf{Solve} + \mathbf{Explore}\end{aligned}$$

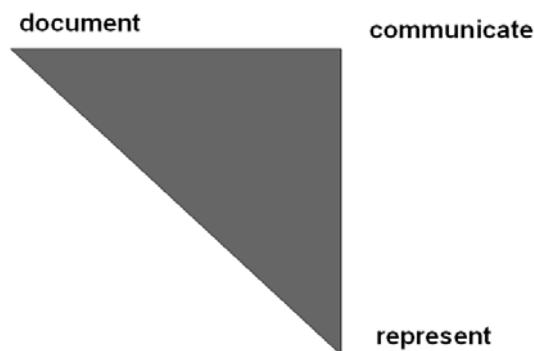
Here is a visualization of the *Automate* triangle¹:



Connection also comprises three archetypes based on what to connect with what, notably *representation*, *documentation*, and *communication*. *Representation* is about connecting models with models, such as connecting an algebraic model (an expression) with a graphic model (a graph) or a numeric model (a table). *Documentation* is about connecting models with humans, such as writing a paper on how a problem was solved. *Communication* is about connecting humans with humans, such as having students work in pairs or groups.

$$\mathbf{Connect} = \mathbf{Represent} + \mathbf{Document} + \mathbf{Communicate}$$

Here is a visualization of the *Connect* triangle²:



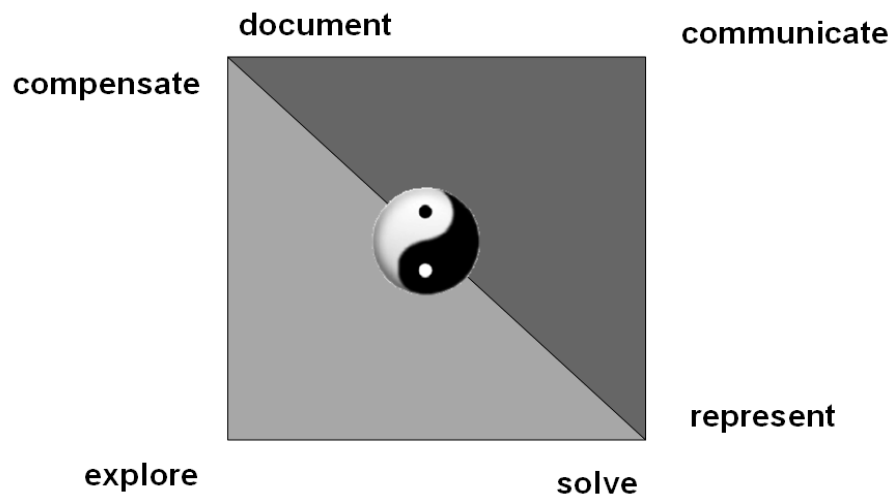
¹ The *Yang* triangle usually points upwards.

² The *Yin* triangle usually points downwards.

Putting these two triangles next to each other yields a picture that I call the *Yin & Yang of Teaching and Learning Mathematics*:

automate

connect



This picture shows six archetypes that we encounter in the context of teaching and learning mathematics (and related subjects). The benefit of this model is to allow for a better understanding of how to best integrate technology into mathematics education.

Before we go through each of the six archetypes and discuss the various roles that technology, in particular computer algebra systems (CAS), can play for each we present an additional picture that is helpful for understanding the benefit of using technology for both mathematics and mathematics education.

